COMOD 2018

COASTAL OCEAN MODELLING WORKSHOP

Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling

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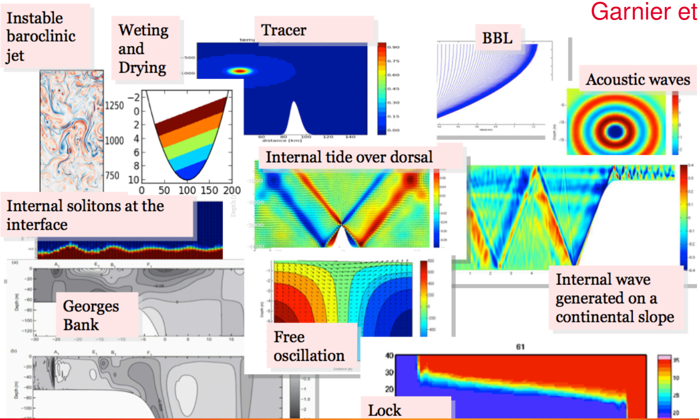
Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)

Funded by the french national research agency (9 postdocs/engineers)

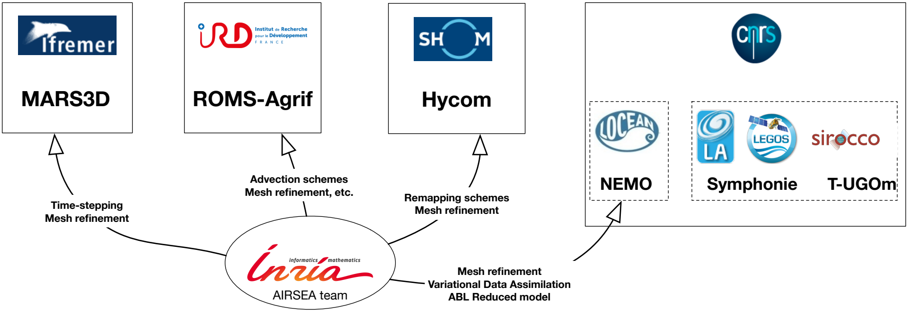
- Intercomparison and evaluation of models
- Improved numerical methods
- Definition of a suite of standardized test cases
- Develop associated diagnostic tools (pyCOMODO tools)

Garnier et al., GMD, in prep.



Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)

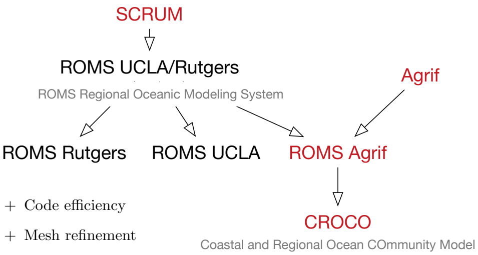
- Modeling groups involved in the project



▷ Most modeling groups had common objectives for coastal applications (e.g. NH option, flexible horiz. and vert. grids, coupling with waves, etc)

→ minimize the duplication of efforts
→ promote interoperability across numerical models (via tools like Oasis, XIOS)

the Croco initiative : an outcome from COMODO



+ Code efficiency
+ Mesh refinement
+ Large community

▷ Roms-Agrif numerical kernel
▷ Online nesting capability via Agrif library
▷ Non-hydrostatic Non-Boussinesq from S-NH
▷ Sediment module from Mars3d
▷ OAW coupling interface (shared w. Nemo)
▷ ALE-type vertical coordinate (ongoing + shared w. Nemo)

Supported by Ifremer, Shom, Cnrs, Ird, Inria

- Croco as a way to sustain the COMODO group
- Complementary to NEMO in terms of target applications

▷ Ifremer : Mars3d $\xrightarrow{\text{transition}}$ Croco
▷ Shom : Hycom $\xrightarrow{\text{transition}}$ Croco

Content: recent developments in CROCO

1. Stability analysis of the mode-splitting and control of numerical dissipation
2. A non-hydrostatic non-Boussinesq algorithm, the how and why
3. Multiresolution simulations using block structured mesh refinement
4. Future perspectives and concluding remarks

1

Stability analysis of the mode-splitting and control of numerical dissipation

Objectives

$\rightarrow$ Spurious ”numerical mixing” not only associated with the space/time discretization of the advection operator

- Objective : characterize the impact of the ”inexact” mode-splitting on the stability of numerical models

Our approach :

1. Stability analysis of the mode splitting technique based on an eigenvector decomposition using the exact barotropic mode
2. Quantify the amount of dissipation required to stabilize the approximative splitting

Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations
$(u_0 = w_0 = 0,\ f_{\text{cor}} = 0)$

$$\begin{cases} \partial_t u + \dfrac{1}{\rho_0}\partial_x \widetilde{p} &= 0 \\ \partial_z \widetilde{p} &= -\widetilde{\rho} g \\ \partial_x u + \partial_z w &= 0 \\ \partial_t \widetilde{\rho} + w \dfrac{d\overline{\rho}}{dz} &= 0 \end{cases}$$

with boundary conditions

$$\begin{aligned} w(z=0) &= \partial_t \eta & \eta \ll H \\ w(z=-H) &= 0 & \text{(flat bottom)} \\ \widetilde{p}(z=0) &= \rho_0 g \eta \end{aligned}$$

Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations

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Sturm-Liouville eigenvalue problem

with $\Lambda = -\dfrac{d}{dz}\left(N^{-2}\dfrac{d}{dz}\right)$

$$\begin{cases} \Lambda M_q(z) &= c_q^{-2} M_q(z) \\ \dfrac{dM_q}{dz}(-H) &= 0 \\ \dfrac{dM_q}{dz}(0) &= -\dfrac{N^2(0)}{g} M_q(0) \end{cases}$$

$\langle M_q, M_m \rangle = \delta_{q,m}$ w.r.t. the inner product $\langle f, g \rangle = \dfrac{1}{H}\displaystyle\int_{-H}^{0} f g \, dz$

Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations
$(u_0 = w_0 = 0,\ f_{\text{cor}} = 0)$

$$\begin{cases} \partial_t u + \dfrac{1}{\rho_0}\partial_x \widetilde{p} &= 0 \\ \partial_z \widetilde{p} &= -\widetilde{\rho} g \\ \partial_x u + \partial_z w &= 0 \\ \partial_t \widetilde{\rho} + w \dfrac{d\overline{\rho}}{dz} &= 0 \end{cases}$$

with boundary conditions

$$\begin{aligned} w(z=0) &= \partial_t \eta & \eta \ll H \\ w(z=-H) &= 0 & \text{(flat bottom)} \\ \widetilde{p}(z=0) &= \rho_0 g \eta \end{aligned}$$

Expand the variables $u, \widetilde{p}$ in the eigenfunctions $M_q(z)$ of Λ

$$\begin{cases} u(x,z,t) &= \displaystyle\sum_{q=0}^{K-1} u_q(x,t) M_q(z) \\ \widetilde{p}(x,z,t) &= g\rho_0 \displaystyle\sum_{q=0}^{K-1} h_q(x,t) M_q(z) \end{cases}$$

to obtain a set of uncoupled systems

$$\begin{cases} \boldsymbol{\partial_t u_q + g \partial_x h_q} &= \mathbf{0} \\ \boldsymbol{\partial_t h_q + \dfrac{c_q^2}{g} \partial_x u_q} &= \mathbf{0} \end{cases}$$

In particular

- $q = 0 \rightarrow$ barotropic mode
- $q > 1 \rightarrow$ baroclinic modes

Usual derivation of the barotropic mode in oceanic models

(e.g. Blumberg & Mellor, 1987; Killworth et al., 1991)

Starting from the PEs, horizontal velocities are decomposed as

$$u = \left(\frac{1}{H} \int_{-H}^{\eta} u \; dz \right) + u_{3\mathrm{d}} = \overline{u} + u_{3\mathrm{d}}, \qquad \int_{-H}^{\eta} u_{3\mathrm{d}} \; dz = 0$$

Standard barotropic mode (associated to the 2D linearized PEs)

Integrating vertically the continuity and momentum equations we end up with

$$\left\{ \begin{array}{rcl} \partial_t \eta + \partial_x (H \overline{u}) & = & 0 \\ \partial_t \overline{u} + g \partial_x \eta & = & -\frac{1}{\rho_0} \partial_x \left(\frac{1}{H} \int_{-H}^{0} p_h \; dz \right) \end{array} \right.$$

Remark : the red term is a ”slow” term absent from the normal mode analysis.

$\rightarrow$ In practice it is kept frozen during the barotropic integration

Interpretation in terms of normal modes

The underlying assumption is that the barotropic mode is depth-independent

$$M_0^\star(z) = 1, \qquad u_0^\star(x,t) = \overline{u}, \qquad \int_{-H}^{0} M_q^\star(z)dz = 0 \ (q \geq 1)$$

The orthogonality condition between modes is lost

- $\rightarrow$ Exact normal mode decomposition $\rightarrow K$ independent systems
- $\rightarrow$ Depth-independent barotropic mode assumption $\rightarrow K-1$ independent systems (for $q \geq 1$) + a system which includes contributions from all modes

Consequences of the depth-independent assumption

1. Some (fast) barotropic contributions are treated as "slow" terms

$$\begin{cases} \partial_t \eta + \partial_x (H\overline{u}) & = & 0 \\ \partial_t \overline{u} + g \partial_x \eta & = & -\dfrac{1}{\rho_0} \partial_x \left(\dfrac{1}{H} \displaystyle\int_{-H}^{0} p_h \, dz \right) \end{cases}$$

$$\frac{1}{H} \int_{-H}^{0} p_h \, dz = \frac{1}{H} \int_{-H}^{0} (p - \rho_0 g \eta) \, dz = \rho_0 g \sum_{q=0}^{K-1} \left\{ \frac{1}{H} \int_{-H}^{0} M_q(z) - M_q(0) \right\} h_q$$

$\Rightarrow$ the contribution of the barotropic mode to the red term is proportional to

For $N^2 = \text{cste}$ $\qquad \dfrac{1}{H} \displaystyle\int_{-H}^{0} M_0(z) - M_0(0) \approx \frac{\varepsilon}{3}, \qquad \varepsilon = \sqrt{\frac{N^2 H}{g}}$

2. The consistency between the barotropic and baroclinic modes must be enforced since they are no longer independent

Time stepping the coupled baroclinic-barotropic system

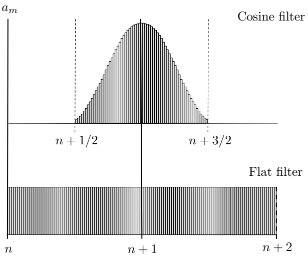
→ Dissipation is necessary to stabilize the time integration procedure because some fast contributions are treated with the slow time-step.

Objective : determine the minimum amount of dissipation required to stabilize the coupled baroclinic-barotropic system integration

Adding dissipation within the barotropic mode

1. Averaging filters (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)

$$\overline{u}^{n+1,f} = \sum_{m=0}^{N_{\text{filter}}} a_m \overline{u}^m \qquad (N_{\text{filter}} \geq N_{\text{split}})$$



under the constraints

$$\sum_{m=0}^{N_{\text{filter}}} a_m = 1, \qquad \sum_{m=0}^{N_{\text{filter}}} a_m \frac{m}{N_{\text{split}}} = 1$$

Several options:

- Flat filter
- Cosine filter

With averaging filters

- the amount of dissipation is somewhat ad-hoc (no quantification of its impact)
- the barotropic integration must go beyond $n+1$
- the barotropic components are not continuous in time

Adding dissipation within the barotropic mode

1. Averaging filters (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)

2. Dissipative time-stepping (e.g. Hallberg, 1997)

In the case of the CROCO ocean model :

$\rightarrow$ a *generalized forward-backward* scheme (*AB3-like, AM4-like*) is used for the barotropic mode (on top of an averaging filter)

$$\begin{aligned}
\eta^{n+1} &= \eta^n - H\Delta t \partial_x \left((3/2+\beta)u^n - (1/2+2\beta)u^{n-1} + \beta u^{n-2} \right) \\
\overline{u}^{n+1} &= \overline{u}^n - g\Delta t \partial_x \left(\delta\eta^{n+1} + (1-\delta-\gamma-\epsilon)\eta^n + \gamma\eta^{n-1} + \epsilon\eta^{n-2} \right)
\end{aligned}$$

- **Revised choice :** no averaging filter + additional constraint for the choice of $(\delta, \gamma, \beta, \epsilon)$ to guarantee stability of the split-explicit formulation

$\Rightarrow$ Minimum level of dissipation to guarantee stability

Impact on nonlinear simulations ?

Linear testcase ($N = 10^{-3}\ \mathrm{s}^{-1}$, $H = 4000\ \mathrm{m}$)

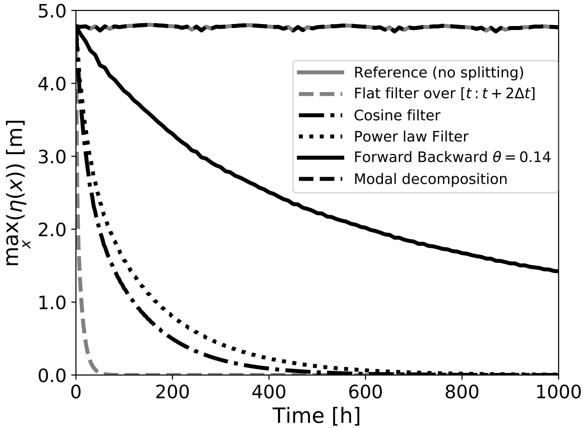
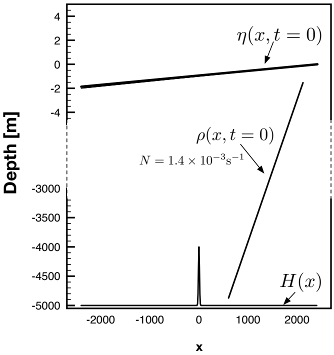


Figure : Time evolution of the maximum absolute value of the free surface elevation. Comparaison of usual filters against a reference solution without splitting.

Internal tide test case

- **Testcase :** nonlinear internal tide generation by topography (Marsaleix et al., 2008)
  - 2D x-z, closed boundaries, $L = 4800\text{km}$
  - Start from rest + linear stratification
  - ▷ Barotropic seiche at a 12-h period
  - ▷ Production of internal waves over the ridge



In the nonlinear case there are other sources of instability :

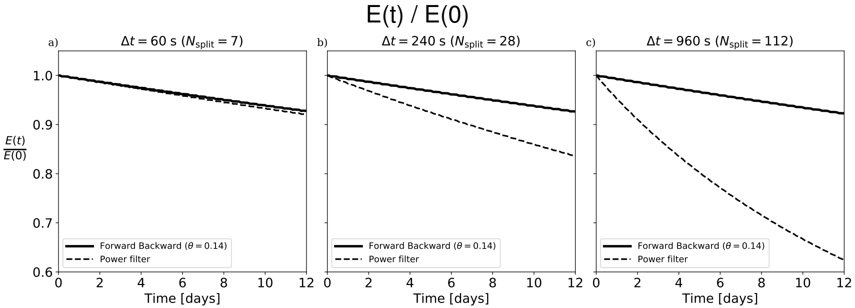
- The rhs of the barotropic equations is integrated from $-H$ to η
- Aliasing errors when the advective terms are added

Internal tide test case

Barotropic mechanical energy

$$E(t) = \int_0^L \frac{\rho_0}{2}(H+\eta)\overline{u}^2 \,\mathrm{dx} + \int_0^L \frac{\rho_0}{2} g\eta^2 \,\mathrm{dx}$$

- Comparison of standard ROMS vs modified CROCO scheme (for a fixed baroclinic time-step)



Summary & perspectives on mode splitting analysis

- ▷ The barotropic mode is traditionally considered as depth independent which is an assumption (e.g. work of Higdon, Bennett, de Szoeke)

- ▷ CROCO approach :

→ Provide a general framework for the stability analysis of the mode splitting approach

→ The framework allow the design of efficient 2D/3D time stepping algorithms

→ Approach implemented in standard Croco oceanic model

- ▷ Perspectives :

→ Transfer toward operational centers (in progress with Mercator-Ocean)

Demange J., L. Debreu, F. Lemarié, P. Marchesiello, E. Blayo: *Stability analysis of split-explicit free surface ocean models: implication of the depth-independent barotropic mode approximation*, submitted to J. Comp. Phys.

2

A non-hydrostatic non-Boussinesq algorithm, the how and why

Classical formulation of a non-hydrostatic oceanic model

2D linearized Non hydrostatic (Boussinesq) equations

$$\begin{aligned}
\partial_t u &= -\partial_x p / \rho_0 \\
\partial_t w &= -\left(\partial_z p - \rho g\right) / \rho_0 \\
\partial_x u + \partial_z w &= 0 \\
\rho &= \rho_{\mathrm{bq}}(\theta, S, -\rho_0 g z)
\end{aligned}$$

Pressure decomposition :

$$p = p_a + p_H + q, \qquad p_H = \rho_0 g \eta + g \int_z^0 (\rho_{\mathrm{bq}} - \rho_0) dz'$$

Boundary conditions :

$$\partial_t \eta = w(0), \qquad w(-H) = 0, \qquad p(z=0) = \rho_0 g \eta \quad \rightarrow \quad q(z=0) = 0$$

q : non-hydrostatic pressure which cancels the divergent part of velocity field

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

$$
\begin{aligned}
\partial_x u &+ \partial_z w = 0 \\
\partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w &= -\partial_z q/\rho_0 \\
\\
\partial_t \eta &= w(0) = -H\partial_x \overline{u}
\end{aligned}
$$

Implicit algorithm

1. Advance η with $\overline{q}^\star = 0$ or $\overline{q}^\star = \overline{q}^n$

$$\frac{\eta^{n+1} - \eta^n}{\Delta t} - gH\Delta t \partial_x^2 \eta^{n+1} = -H\partial_x \overline{u}^n + \frac{\Delta t H}{\rho_0}\partial_x^2 \overline{q}^\star$$

2. Compute provisional velocity field

$$\widetilde{u}^{n+1} = u^n - \Delta t g \partial_x \eta^{n+1}, \qquad \widetilde{w}^{n+1} = w^n$$

3. Solve $\Delta q = \frac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$

4. Correct velocity field to remove divergent part

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

But in the end : $\frac{\eta^{n+1} - \eta^n}{\Delta t} \neq -H\partial_x \overline{u^{n+1}}$

$\rightarrow$ constancy preservation for tracers is lost + 2D/3D inconsistencies

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

$$
\begin{aligned}
\partial_x u \quad &+ \quad \partial_z w = 0 \\
\partial_t u \quad &= \quad -g\partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w \quad &= \quad -\partial_z q/\rho_0 \\
\\
\partial_t \eta \quad &= \quad w(0) = -H\partial_x \overline{u}
\end{aligned}
$$

Implicit algorithm (Casulli; 1999)

1. Compute provisional η with $\overline{q}^\star = 0$ or $\overline{q}^\star = \overline{q}^n$

$$\frac{\widetilde{\eta}^{n+1} - \eta^n}{\Delta t} - gH\Delta t \partial_x^2 \widetilde{\eta}^{n+1} = -H\partial_x \overline{u}^n + \frac{\Delta t H}{\rho_0}\partial_x^2 \overline{q}^\star$$

$$\cdots$$

5. Correct free-surface to satisfy B.C.

$$\frac{\eta^{n+1} - \eta^n}{\Delta t} = -H\partial_x \overline{u^{n+1}}$$

Still 2D/3D inconsistencies (may require a decrease of $\Delta t_{3\mathrm{D}}$)

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

$$\begin{aligned}
\partial_x u &+ \partial_z w = 0 \\
\partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w &= -\partial_z q/\rho_0 \\
\\
\partial_t \eta &= w(0) = -H\partial_x \overline{u}
\end{aligned}$$

Split-explicit algorithm $0 \leq m \leq N_{\text{split}} - 1$

1. Advance η and $\overline{u}$ with $\overline{q}^{\star} = 0$ or $\overline{q}^{\star} = \overline{q}^{n}$

$$\left\{ \begin{aligned} \overline{u}^{m+1} &= \overline{u}^{m} - g(\delta t)\partial_x \eta^{m} - \frac{\delta t}{\rho_0}\partial_x \overline{q}^{\star} \\ \eta^{m+1} &= \eta^{m} - \delta t H \partial_x \overline{u}^{m+1}, \end{aligned} \right.$$

2. Compute provisional fields $\widetilde{u}^{n+1}$ and $\widetilde{w}^{n+1}$
3. Correct $\widetilde{u}^{n+1}$ to enforce $\overline{\widetilde{u}^{n+1}} = \overline{u}^{n+1}$
4. Solve $\Delta q = \dfrac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$
5. Correct velocity field to remove divergent part

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

However : $\overline{u}^{n+1} \neq \overline{u^{n+1}}$

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

$$\begin{aligned} \partial_x u &+ \partial_z w = 0 \\ \partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\ \partial_t w &= -\partial_z q/\rho_0 \\ \\ \partial_t \eta &= w(0) = -H\partial_x \overline{u} \end{aligned}$$

Split-explicit algorithm $0 \leq m \leq N_{\text{split}} - 1$

1. Advance η and $\overline{u}$ with $\overline{q}^\star = 0$ or $\overline{q}^\star = \overline{q}^n$

$$\begin{cases} \overline{u}^{m+1} = \overline{u}^m - g(\delta t)\partial_x \eta^m - \frac{\delta t}{\rho_0}\partial_x \overline{q}^\star \\ \eta^{m+1} = \eta^m - \delta t H \partial_x \overline{u}^{m+1}, \end{cases}$$

2. Compute provisional fields $\widetilde{u}^{n+1}$ and $\widetilde{w}^{n+1}$
3. Correct $\widetilde{u}^{n+1}$ to enforce $\overline{\widetilde{u}^{n+1}} = \overline{u}^{n+1}$
4. Solve $\Delta q = \dfrac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$
5. Correct velocity field to remove divergent part

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

However : $\overline{u}^{n+1} \neq \overline{u^{n+1}}$

Solution 1 : change boundary condition on q to $\partial_z q|_{z=0} = 0$

$$\Rightarrow \overline{u}^{n+1} = \overline{\widetilde{u}^{n+1}} = \overline{u^{n+1}}$$

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

$$\begin{aligned} \partial_x u &+ \partial_z w = 0 \\ \partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\ \partial_t w &= -\partial_z q/\rho_0 \\ \\ \partial_t \eta &= w(0) = -H\partial_x \overline{u} \end{aligned}$$

Split-explicit algorithm $0 \leq m \leq N_{\text{split}} - 1$

1. Advance η and $\overline{u}$ with $\overline{q}^\star = 0$ or $\overline{q}^\star = \overline{q}^n$

$$\begin{cases} \overline{u}^{m+1} = \overline{u}^m - g(\delta t)\partial_x \eta^m - \frac{\delta t}{\rho_0}\partial_x \overline{q}^\star \\ \eta^{m+1} = \eta^m - \delta t H \partial_x \overline{u}^{m+1}, \end{cases}$$

2. Compute provisional fields $\widetilde{u}^{n+1}$ and $\widetilde{w}^{n+1}$
3. Correct $\widetilde{u}^{n+1}$ to enforce $\overline{\widetilde{u}^{n+1}} = \overline{u}^{n+1}$
4. Solve $\Delta q = \dfrac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$
5. Correct velocity field to remove divergent part

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

However : $\overline{u}^{n+1} \neq \overline{u^{n+1}}$

Solution 2 : reset $\overline{u}^{n+1} = \overline{u^{n+1}}$ and $\frac{\eta^{n+1} - \eta^n}{\Delta t} = -H\partial_x \overline{u^{n+1}}$
$\Rightarrow$ loose the gain in accuracy of the split-explicit algorithm

Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode and Poisson pressure equation

Pressure correction / Pressure projection method

- In generalized vertical coordinates, the Poisson pressure equation is hard to solve (25 diagonals, non symmetric)
- Mode splitting: choices
  - ~~relax the compatibility of 2D/3D fluxes~~ (large source of instabilities)
  - relax the true boundary condition $q(z=0)=0$
  - relax the accuracy of the 2D integration (for split-explicit algorithms)
- Likely to require to decrease the baroclinic time-step compared to hydrostatic simulations

Pseudo-compressible approach (Auclair et al., 2017)

2D linearized NH Non-Boussinesq equations

Taylor expansion of density field (with $\frac{\partial \rho}{\partial p} = c_s^{-2}$)

$$\begin{aligned}
\rho = \rho(\theta, S, p) &= \rho_{\mathrm{bq}}(\theta, S, p_{\mathrm{ref}}) + \underbrace{\frac{\partial \rho}{\partial p}\delta p}_{\delta\rho} + \mathcal{O}(\delta p^2) \\
\partial_t u &= -\partial_x p / \rho_0 \\
\partial_t w &= -\left(\partial_z p - \rho g\right)/\rho_0 \\
\partial_t \delta\rho &= -\rho_0 (\partial_x u + \partial_z w)
\end{aligned}$$

Pressure decomposition :

$$p = p_a + p_H + c_s^2 \delta\rho, \qquad p_H = \rho_0 g \eta + g \int_z^0 (\rho_{\mathrm{bq}} - \rho_0) dz'$$

Boundary conditions :

$$\partial_t \eta = w(0), \qquad w(-H) = 0, \qquad p(z=0) = \rho_0 g \eta \quad \rightarrow \quad \delta\rho(z=0) = 0$$

Pseudo-compressible approach (Auclair et al., 2017)

Homogeneous linearized equations

$$
\begin{aligned}
\partial_t u &= -g\partial_x \eta - c_s^2 \partial_x \delta\rho \\
\partial_t w &= -c_s^2 \partial_z \delta\rho \\
\partial_t \delta\rho &= -\rho_0 (\partial_x u + \partial_z w) \\
\\
\partial_t \eta &= w|_{z=0} \\
w|_{z=-H} &= 0 \\
\delta\rho|_{z=0} &= 0
\end{aligned}
$$

Semi-implicit forward-backward

$$
\begin{aligned}
u^{m+1} &= u^m - \delta t \left(g\partial_x \eta^m + c_s^2 \partial_x \delta\rho^m \right) \\
w^{m+1} &= w^m - \delta t c_s^2 \partial_z \left(\delta\rho^{m+\theta} \right) \\
\delta\rho^{m+1} &= \delta\rho^m - \rho_0 \delta t \left(\partial_x u^{m+1} + \partial_z w^{m+\theta} \right) \\
\eta^{m+1} &= \eta^m + \delta t (w|_{z=0})^{m+\theta}
\end{aligned}
$$

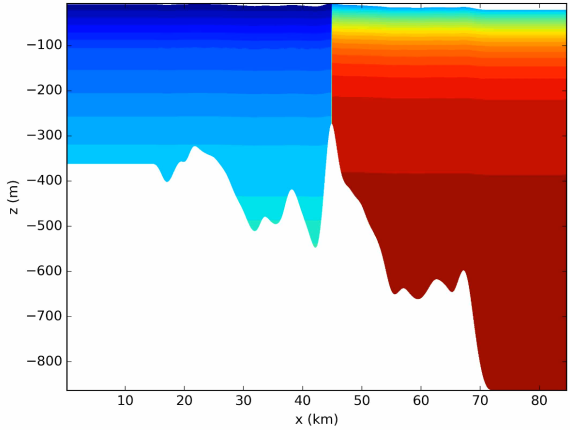
In practice :

- c_s is chosen such that: $\varepsilon = \dfrac{\sqrt{gH}}{c_s} \ll 1$
- The acoustic waves are integrated, in a split-explicit free surface approach, at the same level (i.e. with the same time step) than the barotropic mode.

Advantages of the pseudo-compressible approach (Auclair et al., 2017)

- Surface waves ”feel” the NH effects
- Main limiting CFL condition : horizontal acoustic waves propagation
  $\rightarrow$ may require to decrease the barotropic time-step
- This approach allows internal waves with high order divergence - pressure gradient computation
- Scales well with the resolution
- Compressibility effects physically significant ?

A semi-realistic application (Gibraltar strait)



Simulation characteristics

- NH-NBQ Croco model
- $\Delta x = 150$ m
- 40 vertical layers
- 5-days simulation
- Forced by tides at boundaries

$\rightarrow$ F. Auclair, L. Bordois

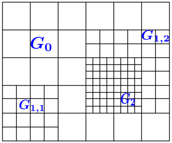
$\Rightarrow$ Robust enough to be used for arbitrary realistic simulations

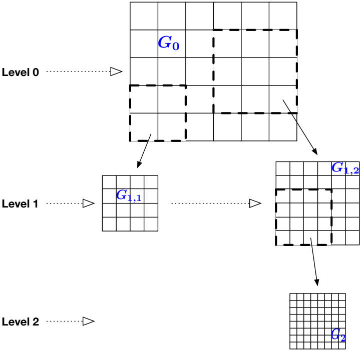
3

Multiresolution simulations using block structured mesh refinement

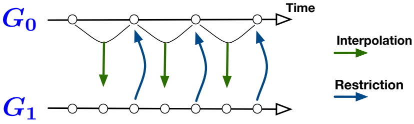
Mesh refinement in structured grid models

Berger & Oliger algorithm

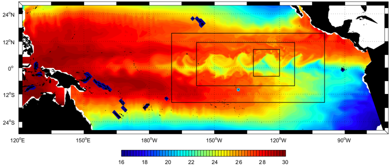




- Time integration



- Possibility to locally adjust the physics, the numerics, the geometry, etc.

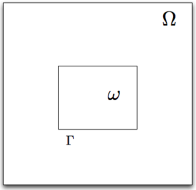


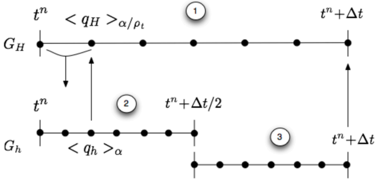
Marchesiello et al., 2012, OM

"Standard" nesting in Croco

- Full two-way coupling (i.e. at the barotropic time-step level)
  - $\rightarrow$ model solution unaffected by nesting when the refinement coefficient is one.
- Local space and time refinement (unlimited number of grids)
- Fully conservative (volume and tracer via refluxing)

- Implemented via the AGRIF library `http://agrif.imag.fr/`



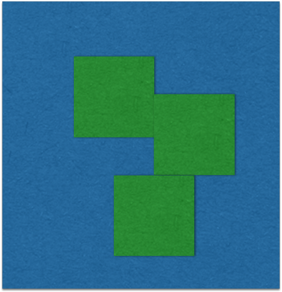


Debreu et al., *Two-way nesting in split-explicit ocean models: Algorithms, implementation and validation*, Ocean Model. (2012)

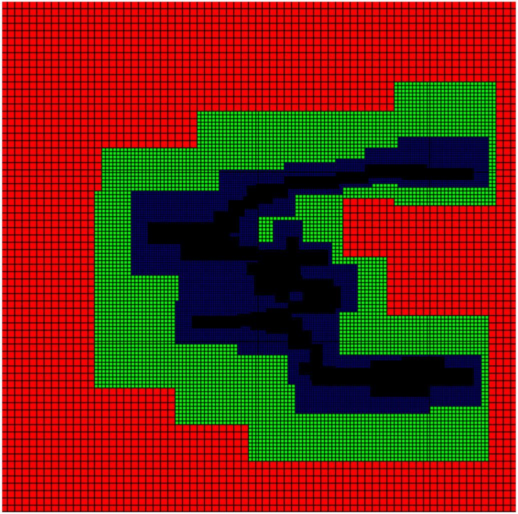
From nesting to truly multiresolution

What is needed for multiresolution ?

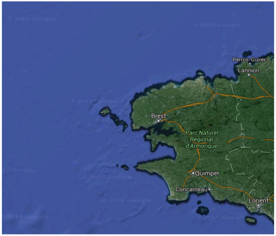
- Neighboring/overlapping grids connection
- Bathymetry smoothing
- Refinement criterion
  - e.g. distance to the coast
- Load balancing on parallel computers
- Inputs / Outputs and visualization
  - Inputs : online interpolation
  - Outputs: Boxlib Format
  - Visualization : VisIt `https://visit.llnl.gov`



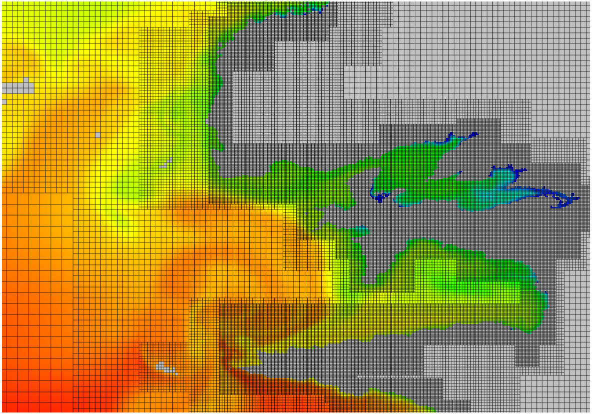
A realistic application (westernmost tip of Brittany)



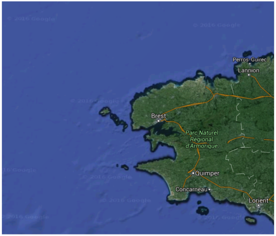
From 2.8 km to 350 m



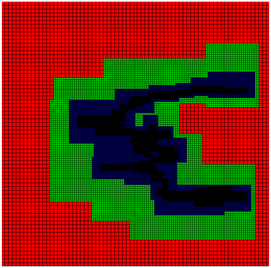
A realistic application (westernmost tip of Brittany)

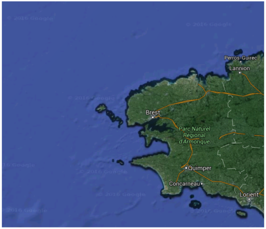


From 800 m to 100 m



A realistic application (westernmost tip of Brittany)

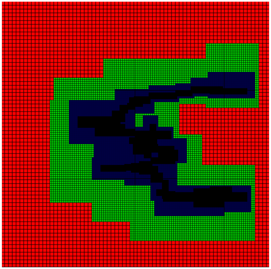


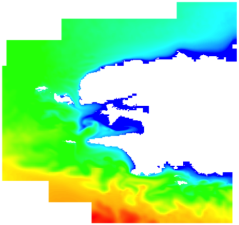


4 levels of refinement

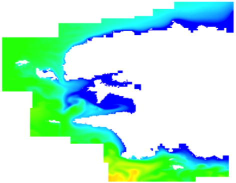
- **Grid resolutions**
  - Level 1 : 2.8 km
  - Level 2 : 1.4 km
  - Level 3 : 700 m
  - Level 4 : 350 m
- **Area covered by the different levels**
  - Level 1 : 100%
  - Level 2 : 43%
  - Level 3 : 18%
  - Level 4 : 7%
- **Number of grids per levels**
  - Level 1 : 1
  - Level 2 : 8
  - Level 3 : 20
  - Level 4 : 46

A realistic application (westernmost tip of Brittany)



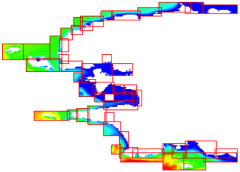


Level 2



Level 3

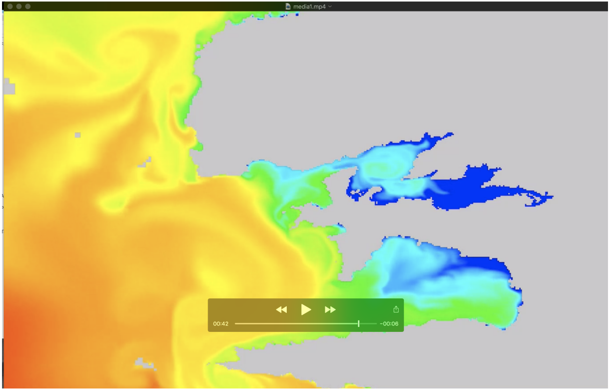
- Minimum number of grid points per grids : 256 (16x16)
- Percentage of land points at the finer level : 20%
- Cost of intergrid operations : 15%



Level 4

A realistic application

Sea surface temperature (multiresolution from $\Delta x = 800\ \mathrm{m}$ to $\Delta x = 100\ \mathrm{m}$)



Simulation characteristics

- PE Croco model
- Refinement of 2
- 4 levels of refinement

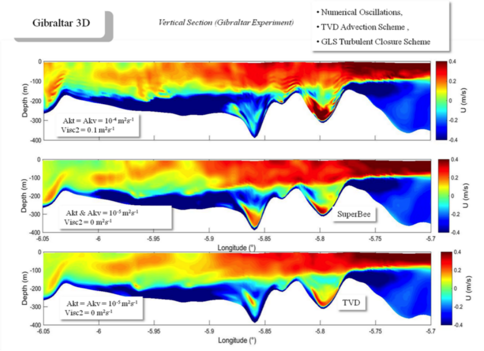
- Tides
- Wetting & drying

4

Future perspectives and concluding remarks

Some future challenges for Croco

- (Non-hydrostatic) - (Hydrostatic) coupling (for OBCs and nesting)
- 3D closure (LES vs MILES)
- Relative role of truncation errors vs subgrid terms
- Coupling with external components
- Perpetual revision of time-stepping and numerical schemes



Feedback from model intercomparison

Experience feedback from COMODO project

Positive aspects

- Inter-disciplinary collaborations around specific scientific questions
- Useful for code debugging and checking of numerical implementation
- Helpful to prioritize future code developments (e.g. Ilicak et al., 2011)
- Dynamic effect on the community with experience-sharing

Difficulties/limitations

- Difficult to define reference solutions
- ”Good” solutions obtained for the wrong reasons
- Focused on dynamical core, no parameterization
- Generally simple geometries
- Common format for structured and unstructured grid models

An illustration from COMODO project

▷ Computational and numerical aspects

Numerical and computational aspects

Stability range

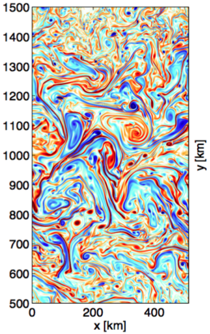
Theoretical stability limit (e.g. Lemarié et al., 2015)

	NEMO	CROCO	Mars3D
Internal gravity waves	0.46	0.85	2
External gravity waves	0.46	0.9	∞
Advection	0.46	0.87	1

Baroclinic jet tetcase

$\rightarrow$ time-step constrained by IGW propagation

	NEMO	CROCO	Mars3D
Δt_{3D}[s]	200	340	**320**
Δt_{2D}[s]	3	6.1	320



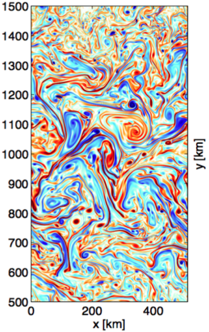
Baroclinic jet testcase (Soufflet et al., 2016)

$\rightarrow$ Development of offline diagnostics to compute stability limits within the Pycomodo tools

Numerical and computational aspects

Sequential performance (Intel VTune profiler)

	NEMO	CROCO	Mars3D
Memory size	2Gb	800Mb	1,4Gb
Number of instructions	5.5 bil.	3.3 bil.	**13.9 bil.**
Vectorization (%)	40	**78**	45
Cache bound[1] (%)	14	14	**71**
FP Arith./Mem. Rd Instr.[2]	0.56	**1.43**	0.62
Execution time (s)	**609**	**160**	**686**



[1] percentage of execution time spent in cache memory accesses
[2] floating point arithmetic instructions per Memory Read or Write

→ e.g. results used as a basis for the NEMO development strategy

- COMODO Project: `http://www.comodo-ocean.fr/`
  (now an online shopping site !)
- Pycomodo tools: `http://pycomodo.forge.imag.fr/`

`https://www.croco-ocean.org/`